Letter to Prospective Customers, from Jacob McCoy and Ben Wacker:

The ACE manual was designed with the intent of clarifying complex text (and problems) with explanations in plain-English. This is accomplished via clear and concise summaries of each chapter, author's commentaries for the most difficult ("Greek") material, ACE original problems, and more in-depth explanations and answers to some of the problems that are in the book. The study guide has calculations and practice problems integrated with the outline to facilitate learning.

ACE is committed to providing many calculation-type problems. Please contact us with any questions at <u>*acestudyguide@yahoo.com*</u>

ACE CSP Canada Study Guide – Sample Chapter

Life Insurance Products and Finance Chapter 15 Stochastic Modeling

1) Introduction
   a) Deterministic vs. stochastic modeling
      i) Deterministic: Use a single set of predetermined assumptions.
      ii) Stochastic: Use multiple sets of assumptions that are determined by a probability distribution and random variables
   b) Can use stochastic assumptions if uncertain about future values (interest rates or mortality)
   c) Strategies to reduce the effect of adverse interest rates
      i) Use conservative assumption
      ii) Offer products that adjust benefits as interest rates change
      iii) Match assets and liabilities
   d) Stochastic modeling can help a company better understand its risk
   e) Stochastic modeling produces a distribution of possible results

2) Overview of Stochastic Modeling
   a) Uses of Stochastic Models
      i) Asset liability management
      ii) Allows for a more complete understanding of company's risks
      iii) Price embedded options and guarantees
   b) Steps in stochastic modeling
      i) Select a distribution function
         (1) This will vary by assumption
         (2) Common distribution functions are the binomial, normal, and log-normal.
      ii) Generate random numbers
      iii) Stochastically generate sets of variables
         (1) If mortality is being modeled a random number will be generated for each cell, in each time period

(2) For interest rates usually just generate random variables for a couple key rates in each time period, other maturities and asset classes can be derived from these rates.

iv) Calculate results for each set of interest rates/mortality rates

(1) Some assumptions should be dynamically calculated. For example, if interest rates are low, crediting rate should also be low.

(2) Possible assumptions that interest rates could impact:

(a) Credited rates
(b) Dividends
(c) Lapse Rates
(d) Sales
(e) Premiums/COI
(f) Asset calls and prepayments

3) Random Variables

a) Steps to create random variables

i) Pick a distribution function

ii) Calculate a random number that is evenly distributed between 0.000 and 1.000

iii) Calculate the random variable given the random number, such that

(1) F(x-1) < S <= F(x)

b) Binomial Distribution

i) Commonly used to model mortality

ii) N= Number of trials

iii) X = Number of successes

iv) P = Probability of success

v) $\mu = np$

vi) $\sigma^2 = np(1-p)$

c) Normal Distribution

i) If N is sufficiently large, can use binomial to approximate the normal

$$Z_n = \frac{X_n - \frac{1}{2}N}{\frac{1}{2}N^{\frac{1}{2}}}$$

ii) Z_n approximates the normal distribution, can use this if N>=30

4) Stochastic Mortality

a) Stochastic mortality models usually assume that all lives are independent but that is not always true

i) An insured may have more than one policy

ii) Catastrophic events

iii) Lonely heart syndrome or joint accident risk

b) Seriatim Stochastic modeling
   i) Model one policy at a time, one time period at a time
   ii) Steps
      (1) Generate random number S_1
      (2) If $S_1 <= q_d$ then policy is terminated by death (q_d=1)
      (3) If policy is not termed by death then generate another random number S_2
      (4) If $S_2 <= q_w$ then policy is terminated by lapse (q_w =1)
      (5) If policy doesn't lapse or die then the process is repeated next time period
   iii) Advantage -- this is how actual policies behave
   iv) Disadvantage -- it takes a lot of random numbers and computer power

c) Alternative to stochastic modeling
   i) Model largest policies using seriatim stochastic method
   ii) Model the volatility in remaining block by a normal distribution
      (1) Mean of Normal = Total face amount * [Avg q(t)]
      (2) Variance of Normal = Number of policies * q(t) * (1 - q(t)) * $(\text{Average DB})^2$
      (3) Generate a random number and calculate benefits paid
   iii) Assumes identical policies

d) Binomial stochastic mortality modeling
   i) When seriatim method is not feasible, binomial method is a good alternative
   ii) Often used when policies are grouped
   iii) Able to generate one random number to determine a stochastic outcome for n policies at once
   iv) Applying the binomial distribution
      (1) Determine q_d, q_w, and N for the cell in a given time period
      (2) Create the CDF or F(x) for the number of deaths (x) in the period. This will require creating a table of f(x) and F(x).
      (3) Generate a random number S_1
      (4) If $F(x-1) < S_1 <= F(x)$ then X is the number of deaths and $q_d = X/N$
      (5) Generate a random number S_2
      (6) If $F(y-1) < S_2 <= F(y)$ then Y is the number of lapses and $q_w = Y/(N-X)$
      (7) The number of policies in-force at the beginning of the next time period is (N – X – Y)
   v) Calculating the distribution function
      (1) $f(x) = {}_nC_x\, q^x\, (1-q)^{n-x}$
      (2) Bring a calculator to the exam that will solve for ${}_nC_x$
   vi) Fratio
      (1) Often is easier to calculate f(x) once using above formula, then use fratio to calculate f(x+1)
      (2) f(x) = f(x-1) * fratio(x)

$$fratio = \frac{q}{1-q} \times \frac{n-x+1}{x}$$

vii) Cumulative distribution function
  (1) The cumulative distribution function (CDF) is the summation of the probability distribution function (PDF).

$$F(s) = \sum_{0 to s} f(x)$$

5) Stochastic interest rates
   a) The more scenarios created, the more credible the results
   b) Stochastic modeling of interest rates is done at the aggregate level
   c) Complexities involved in stochastically modeling interest rates:
      i) Must project an entire yield curve for each period, not just one rate
         (1) Solution: Stochastically generate short term rate and long term rate, and use interpolation to get yield curve
      ii) Must project yield curves for each asset class (Treasuries, bonds, real estate, etc)
         (1) Solution: Stochastically generate rates for government bonds and assume constant spread for other asset classes
      iii) Interest rates are impacted by events such as wars, economic cycles, and oil shortages (This can be difficult to predict or model)
      iv) Interest rates are impacted by the supply and demand for money (This can be difficult to predict or model)

   d) Yield curves
      i) A yield curve is a graph of interest rates for a given time to maturity
      ii) Normal yield curve will be increasing at a decreasing rate
      iii) Inverted yield curves will be decreasing at a decreasing rate (This implies that short term rates will decrease in the future)
      iv) Interpolating yield curves example
         (1) As mentioned previously, a stochastic process will generate a short term rate and a long term rate
         (2) 1-year rate = (1-0.3600) * 90-day rate + 0.3600 * 10-year rate

   e) Interest rate scenarios
      i) More complexities
         (1) Interest rates are correlated from one period to the next
         (2) Different maturities within the same yield curve are correlated (90-day rate and 10-year rate)
      ii) The three methods presented will address these complexities differently

      iii) Arbitrary method
         (1) Not stochastic, rather, it's scenario testing
         (2) Doesn't involve probabilities
         (3) Results have limited value

      iv) Probabilistic method
         (1) Assume every yield curve is defined by level and slope

(2) Level = 10-year rate
(3) Slope = ratio of 90-day rate to 10-year rate
(4) Use historical information to develop a grid of probabilities
(5) Example Table 15.5.5
   (a) Use table of probabilities and randomly generated numbers to get level and slope of yield curves for each period
   (b) Period #1 Level: Random number = 0.29419
   (c) F (-0.25%) < Random Number <= F (0.00%)
   (d) X = 0.00% (No Change in level)
   (e) Period #1 Slope: Random number = 0.17107
   (f) F(0.60) < Random Number <= F(0.75)
      (i) Convert PDF to CDF by summing
   (g) X = 0.75 (No Change in slope)

v) Successive ratios method
(1) Assumes the natural log of the ratio of successive rates is normally distributed -- $\ln(i_{t+1} / i_t)$ is normal
(2) The correlation from one period to another is reflected by modeling successive ratios
(3) The correlation between the 90-day rate and the 10-year rate is reflected by generating random numbers Z1 and Z2.
   (a) 90-day rate depends on Z1
   (b) 10-year rate depends on Z1 and Z2
(4) Use historical information to develop volatility factor and correlation between 90-day rate and 10-year rate.
(5) Steps to successive ratio method
   (a) $i90Day(t+1) = i90Day(t) * e^{Z1 * VolFactor}$
   (b) $Z10Year = Z1 * Corr + Z2 (1 - Corr^2)^{0.5}$
   (c) $i10Year(t+1) = i10Year(t) * e^{Z10Year * VolFactor}$

(6) Example -- Table 15.5.6
   (a) You are suppose to use Jetton's approximation to the normal to get Z1

$$Z1 = \frac{12 - \frac{1}{2}(30)}{\frac{1}{2}(30)^{\frac{1}{2}}} = -1.09545$$

   (b) $Z10Year(1) = -1.09545 = -1.09545 * 1.00 + 1.46059 * (1 - 1^2)^{0.5}$
   (c) $i90Day(t+1) = 6.00\% * e^{-1.09545 * 0.15}$
   (d) $i10Year(t+1) = 8.00\% * e^{-1.09545 * 0.15}$

(7) Advantages of successive ratios method
   (a) Not limited to predetermined level or slope, combinations are infinite
   (b) No need to create large tables or calculate probabilities

(8) Disadvantages of successive ratios method
(a) Can produce crazy interest rates, but this can be corrected by applying a max and min.
(b) No mean reversionary process
(c) Produces more inverted yield curves than expected
(d) Difference between 90-day rate and 10-year rate can become vary large, once again this can be corrected by applying a cap or floor.

6) Effect on liabilities
a) Determining interest rates
i) At the start of each new period in the model need to determine:
(1) Average earned rate on existing assets
(2) Interest rate on new investments
(3) Interest rate available on competing products (Market rates)
(4) Credited rate

b) Modeling market interest rates
i) Market rate should reflect what the competition's crediting rate
ii) If credited rate is near the market rate, then lapse shouldn't be impacted
iii) If credited rates are below the market rate, then lapses will increase
iv) If product doesn't have an explicit credited rate, can use a proxy such as a dividend rate

v) New money vs. Portfolio method
(1) New money method
(a) New deposits are given a "new money rate" to better reflect what new purchase assets are currently earning
(b) The new money rates are updated quarterly or annually, resulting in different parts of the account value earning different rates
(c) Also called the segmentation method
(2) Portfolio method: All of the account value earns the same portfolio rate
(3) Market rate could be the higher of the new money rate and the portfolio rate

c) Modeling credited interest rates
i) Credited rate is a function of four rates:
(1) Earned rate, or portfolio rate
(2) New money rate
(3) Interest rate guarantees
(4) Market rate
ii) If segmentation method is used then the model must be able to track assets, liabilities, and credited rates by segment
iii) The product may have one or more of the following guarantees
(1) Long term guaranteed interest
(2) Short term guaranteed interest

            (3) Bailout rate -- If credited rate drops below bailout rate, then surrender charges are waived.
      iv) Most companies will have a targeted spread (Earned rate less credited rate)
      v) Competitive pressures (low market rates) often prevent company from earning targeted spread
      vi) Credited rate formula
            (1) Most companies have guidelines when setting credited rate, such as:
                  (a) No less than X% of market rate
                  (b) No less than X bps spread
                  (c) Guideline could also consider surrender charges

   d) Modeling the effect on lapses
      i) If market rates are higher than the credited rate, customers will move their money to the competition
      ii) Sensitivity will vary by type of product and duration
      iii) A stochastic model should have a dynamic lapse rate that reflects the impact of:
            (1) Credited rates
            (2) Market rates
            (3) Surrender charges
            (4) Duration
      iv) One possible formula
            (1) $qw(t) = qwBase(t) * (1 + 0.5(100 * Diff)^2)$ –SurrChg%
            (2) Diff = Market rate less credited rate
      v) Lapses should increase and decrease exponentially as market rates and credited rates move further apart

   e) Modeling other product cash flows
      i) Credited rates will impact reserves (CV floor)
      ii) The difference between credited rates and market rates will impact partial withdrawals and premium persistency
      iii) Market rates will impact policy loan utilization, especially if policy loan rates are fixed
      iv) Inflation if often linked to interest rates
      v) High lapses often lead to poor mortality experience (anti-selection)

7) Effect on Assets

   The syllabus skips section 7.

8) Summarizing stochastic results
   a) Profit targets for a stochastic model are different than a deterministic model
      i) Could look at median profit result
      ii) Could look at 25th and 75th percentile
      iii) Could look at weighted average profit

b) Scenarios are often summarized by percentile and graphed to see the range of results.

9) Exercises -- Answers are in the book so I won't reproduce them, but I have included some notes.
   a) 15.1
      i) Find X so that F(X-1) < S <= F(X)
      ii) If S = 0.04224, then X = 7 points.
      iii) If S = 0.04225, then X=14
   b) 15.3
      i) They want you to work backwards
      ii) If Zn = -1.09545, then X=12, and P(X=12) = 0.0770
   c) 15.4 -- You should memorize the formula for fratio, you don't have to derive it.
   d) 15.6 -- I would suggest doing this in excel
   e) 15.11
      i) This could be a part of a larger exam question
      ii) Formula should have a min and max
      iii) The lapses should increase exponentially as the spread between the market rate and credited rate widens
      iv) Should consider surrender charges
   f) 15.13
      i) Bonds: Calls, puts, sinking funds
      ii) Mortgages and CMO: refinancing, or extra payments
         (1) Might have prepayment penalties
      iii) If this was an exam question, you would want to elaborate more on what a call is and what a put is, etc.